Control Parameters Tuning,
Sensitivity and Robustness Evaluation of a PID-Controller

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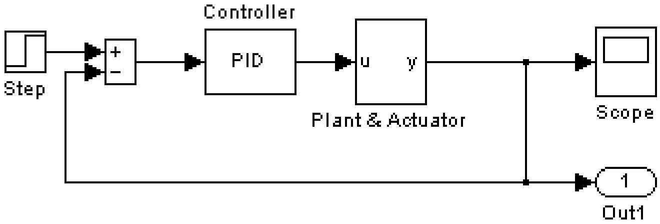
OptiY-File: *...\OptiY\CAE Integration\Matlab\PIDController.opy*

1. Auto-Tuning of Control Parameters

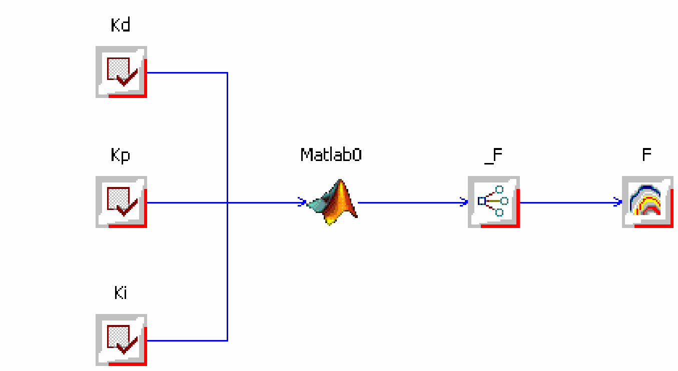
A proportional–integral–derivative controller (PID controller) is a generic control loop feedback mechanism widely used in industrial control systems. A PID controller attempts to correct the error between a measured process variable and a desired set-point by calculating and then outputting a corrective action that can adjust the process accordingly. The PID controller calculation involves three separate parameters: the Proportional Kp, the Integral Ki and Derivative values Kd. The Proportional value determines the reaction to the current error, the Integral determines the reaction based on the sum of recent errors and the Derivative determines the reaction to the rate at which the error has been changing. The weighted sum of these three actions is used to adjust the process via a control element such as the position of a control valve or the power supply of a heating element. By tuning the three constants in the PID controller algorithm the PID can provide control action designed for specific process requirements.



Picture1: PID-Controller

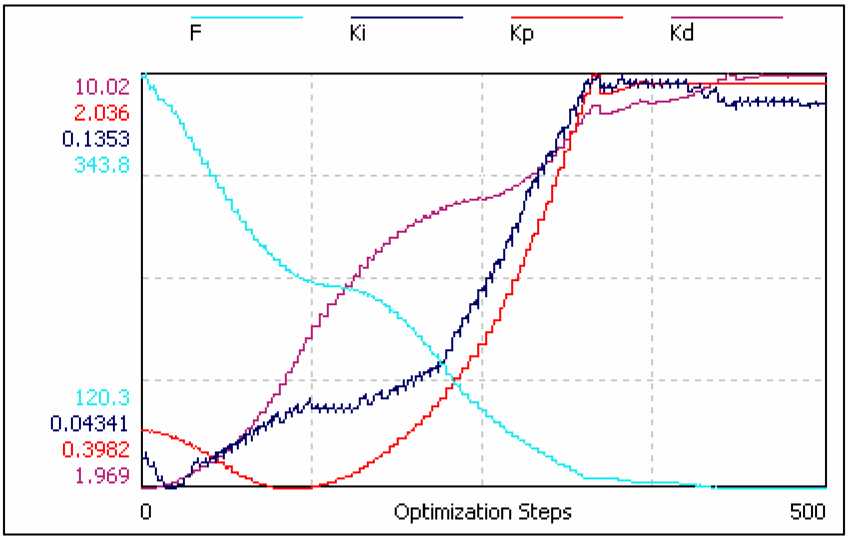
The PID-Controller with a given plant is modelled in Simulink *optsim.mdl* (Picture1). The plant and actuator will be handled detail in the next section. The Controller should bring the plant von position 0 to the position 1 fast and without vibrations. Thereby, the control parameters Ki, Kd, Kp should be tuned to set controller to the point. Control parameters tuning leads to an optimization task. The control parameters will be optimization variables being searched for optimal values to coincide the target signal 1 and measuring signal *y* of the controller. In the matlab-script *callsim.m* performing simulation of the simulink model, the criterion for optimization *F* will be defined:

$$F = \int |y-1| dt \quad (1)$$



Picture2: OptiY-Workflow for Control Parameters Tuning

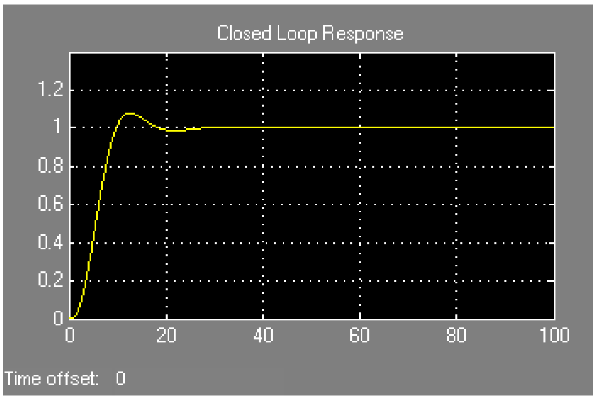
The integral (1) between the target signal 1 and the measuring signal *y* represents the area between both curves. If the area is minimized, both curves will be coincided together as most as possible.
The experiment for the optimization is built In OptiY with the matlab-script *callsim.m* (Picture2). Die control parameters Ki, Kp, Kd are linked with the optimization variables. The criterion is signed to the objective function *F*.



Picture3: Optimization Process

The start values and search range of the optimization variables are set (Table1). The optimization process is done by the standard parameters in OptiY (Hooke-Jeeves-Method). After 500 optimization steps, the objective function F get minimized (Picture3). During optimization, changing of the control parameters can be seen live. The results of the optimization are listed in Picture4, Picture5 and Table1.

Picture4: Before Optimization



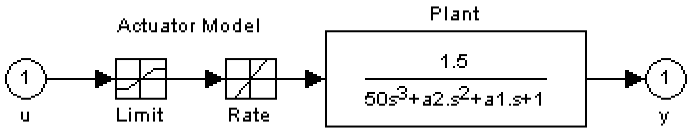
Picture5: After Optimization

Control Parameters	*Start Value*	*Range*	*Optimal Value*
Kp	0.63	[0.1 ; 2]	1.9999
Kd	1.9688	[0.2 ; 10]	9.9999
Ki	0.0504	[0.001 ; 1]	0.128822

Table1: Optimization Data

2. Sensitivity and Robustness Evaluation of the Controller

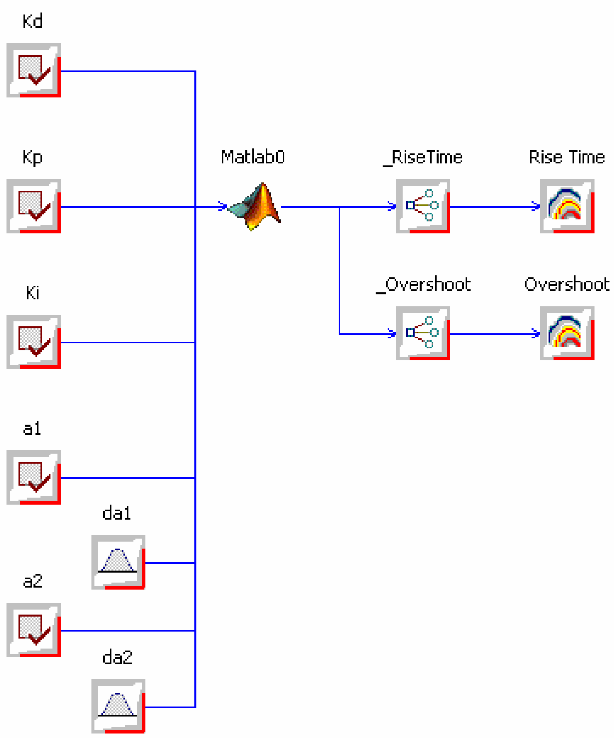
The plant model is a generic machine with an actuator. The mathematical description of the model contains third order differential-equation (Picture6). Any model can be also considered.



Picture6: Plant Model

The tuning of the control parameters is done under ideal conditions for the actuator and plant. In the real-world, all parameters lie under stochastic and nature variability, which represent the uncertainty or randomness for the controller. Following aspects of uncertainties can be considered:

- Changing Loading: mass, moment, etc.
- Environment conditions: different temperature, air humidity, etc.
- Material properties: aging, erosion, friction, etc.
- Manufacturing inaccuracy: geometry tolerances, etc.



Picture7: OptiY-Workflow for Uncertainty Analysis

The plant contains 2 uncertainty parameters a1 and a2, which represent some of the above properties. Following data are used for the probabilistic simulation evaluating sensitivity and robustness of the PID-Controller (Table2). The tolerance values add up to 10% of the nominal values of uncertainty parameters. All uncertainties are normal distributed.

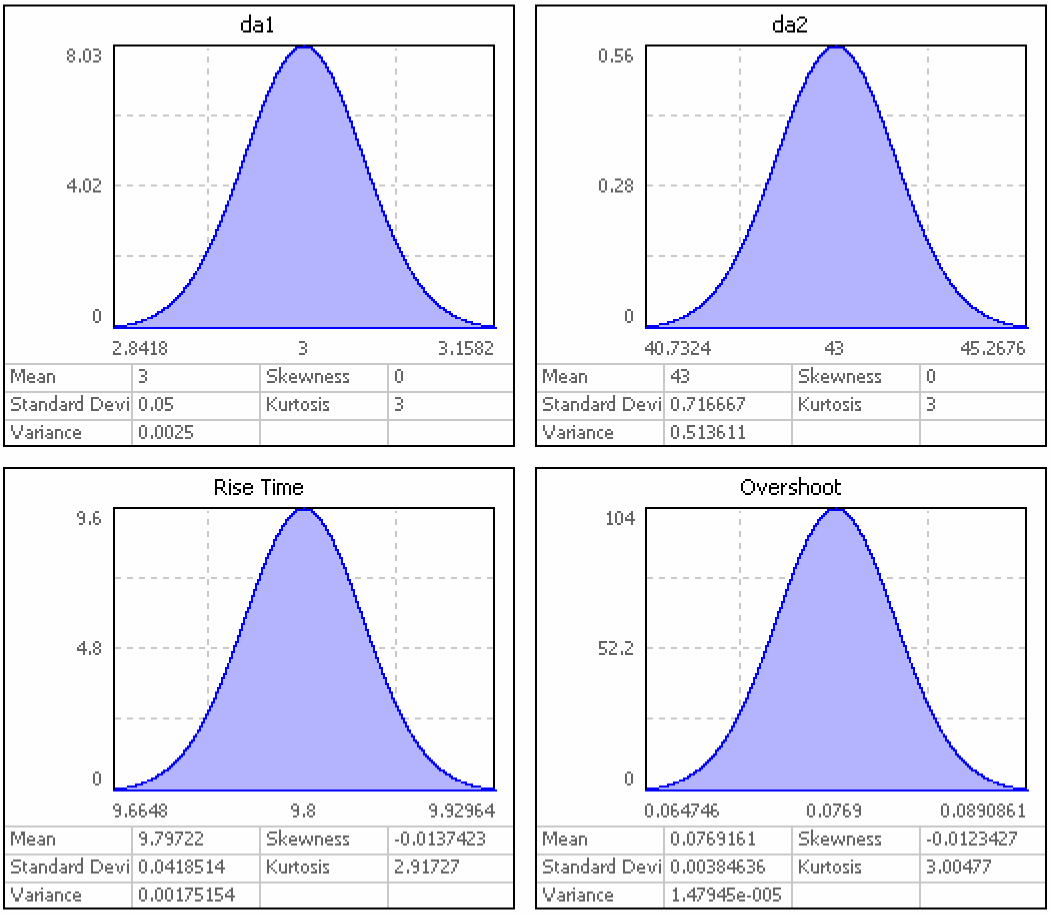
Uncertainty Parameters	*Nominal Value*	*Tolerance Value*	*Distribution*
a1	3	0.3	Normal
a2	43	4.3	Normal

Table2: Uncertainty Data

These uncertainties cause that the PID controller can work not correctly or instable. The questions arise:

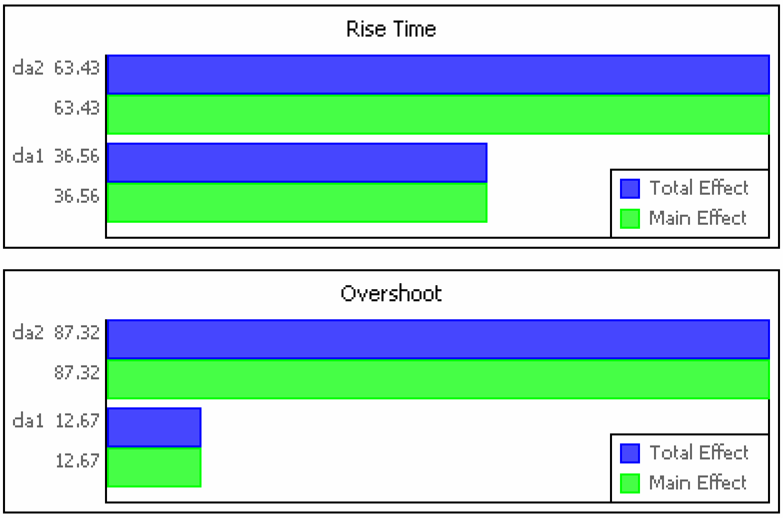
- How robust does the controller work with the given control parameters?
- Which uncertainty parameters cause most the variability of the control goodness?

The control goodness defined as the rise time and overshoot of the PID controller. An experiment is built in OptiY to study these aspects (Picture7). Beside the control parameters Ki, Kd, Kp, there are 2 uncertainty parameter da1 an da2 with its nominal value a1 and a2. The criteria of the experiment are the rise time and overshoot of the controller. The design of experiment is done in OptiY with the second-order-method.



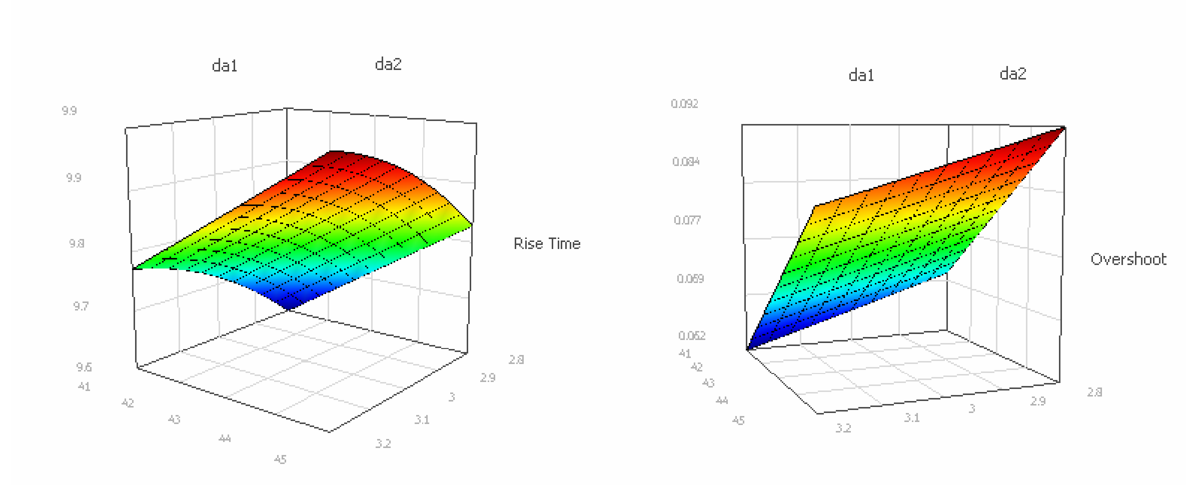
Picture8: Input and Output Distributions

The results present the normal distributions of the rise time and overshoot of the PID controller. The rise time moves between 9.6648 and 9.9296 second. The standard deviation adds up to 0.041 seconds. The mean value lies at 9.79 second. The overshoot changes between 0.065 and 0.089. The standard deviation amounts to 0.0038 and the mean value 0.0769. With all uncertainties a1 and a1 about 10% of nominal values, the controller works very robust and stable. The variability of the control goodness underlies a small and unremarkable range.



Picture9: Global Sensitivities

With the Pareto Chart (Picture9), the global sensitivities of the rise time and overshoot can be considered. The variability of the uncertainty parameter a2 causes most the variability of the control goodness (rise time and overshoot). If the variability of the uncertainty a1 is minimized, the variability of control goodness will be reduced smaller. The controller will work more robust and stable. The total effect and the main effect are the same. That means no interactions between both uncertainty parameters.



Picture10: Response Surfaces

Furthermore, information about the correlation between input (a1, a2) and output (rise time and overshoot) is also available. The response surfaces (Picture10) present a quadratic correlation between uncertainty parameters and the rise time. The correlation between the uncertainties and the overshoot is linear. If the values of the uncertainty parameters are input, the values of the output will be displayed at once without any simulation.